Matrix approach to hypercomplex Appell polynomials

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Dedicated to Professor Francesco A. Costabile on his 70th birthday

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Abstract

Recently the authors presented a matrix representation approach to real Appell polynomials essentially determined by a nilpotent matrix with natural number entries. It allows to consider a set of real Appell polynomials as solution of a suitable first order initial value problem. The paper aims to confirm that the unifying character of this approach can also be applied to the construction of homogeneous Appell polynomials that are solutions of a generalized Cauchy-Riemann system in Euclidean spaces of arbitrary dimension. The result contributes to the development of techniques for polynomial approximation and interpolation in non-commutative Hypercomplex Function Theories with Clifford algebras.

Keywords: hypercomplex differentiability, Appell polynomials, creation matrix, Pascal matrix
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